UNITÀ DIDATTICA: AN INTRODUCTION TO TRIGONOMETRY

a cura di Paolo Delise - I.T.C. Carli - Trieste

Finalità

Presentare agli allievi di un corso per ragionieri le nozioni di base di trigonometria e le caratteristiche fondamentali delle funzioni goniometriche. La trigonometria riveste, per gli allievi ragionieri, una funzione fondamentalmente culturale e ad essi non sono richieste le competenze tecniche che vengono invece richieste agli allievi dei corsi ad indirizzo tecnico o scientifico.

Obiettivi dell'insegnante di matematica

Gli allievi, alla fine dell'unità, saranno in grado di:

- enunciare le definizioni delle funzioni trigonometriche
- risolvere semplici problemi analoghi a quelli presentati nel corso dell'unità
- dato un problema concreto che preveda la triangolazione mediante triangoli rettangoli, risolverlo calcolando le lunghezze dei triangoli rettangoli coinvolti. (non necessariamente acquisito da tutti)

Gli allievi, inoltre, saranno avviati verso:

- collaborare tra di loro mediante i lavori di gruppo

Obiettivi CLIL

Gli allievi, alla fine dell'unità, avranno:

- appreso alcuni nuovi vocaboli della lingua inglese
- fatto uso della lingua inglese per comunicare tra loro e con l'insegnante
- preso appunti in inglese

Organizzazione dell'unità

Tempo	*Il docente*	*Gli allievi*
5 minuti	Espone in inglese lo scopo del progetto e gli obiettivi matematici e clil.	Ascoltano
45 minuti	Presenta il problema guida	Ascoltano e prendono appunti
	Pone le domande. Mentre continua la discussione l'insegnante distribuisce il primo foglio della dispensa (pag.3).	Rispondono alle domande in una discussione guidata
	Espone il contenuto della seconda pagina.	Ascoltano e prendono appunti
	Distribuisce il secondo foglio (pag. 4) dell'unità ed invita gli allievi a rispondere ai quesiti. L'insegnante sollecita gli allievi che non sono intervenuti spontaneamente	Eseguono i calcoli e le misure necessarie e rispondono ai quesiti.
	Prosegue esponendo il contenuto della terza pagina	Gli allievi ascoltano, eseguono i calcoli proposti, intervengono.
	Distribuisce la terza pagina (pag. 5).	
10 minuti	Riassume quanto trattato nell'ora precedente	Ascoltano

Tempo	*Il docente*	*Gli allievi*
40 minuti	Accenna al problema degli altri rapporti trigonometrici.	
		Individuano gli altri rapporti.
	Dà il nome ai rapporti trovati dagli allievi	
	Distribuisce la quarta e la quinta pagina (pagg. 6 e 7)	
	Invita gli allievi a completare la pagina 7	Completano la pagina. Possono confrontarsi, ma in inglese
	Commenta i risultati trovati	
	Presenta i contenuti della sesta pagina (pag. 8).	Ascoltano e prendono appunti
	Invita gli allievi a rispondere alla domanda della sesta pagina. Nel frattempo distribuisce le copie della pagina.	Rispondono
10 minuti	Riassume i temi delle due lezioni precedenti	Ascoltano
40 minuti	Prosegue esponendo il contenuto della settima pagina (pag. 9).	Ascoltano e prendono appunti
	Distribuisce la pagina.	Eseguono il disegno che viene richiesto.
	Presenta la soluzione corretta e la commenta	Intervengono per chiedere chiarimenti se hanno sbagliato.
	Presenta il radiante; verso la fine della lezione distribuisce l'ottava pagina (pag. 10).	Prendono appunti , sollecitati dal docente durante l'esposizione, intervengono a rispondere ai quesiti che egli pone durante l'esposizione.
	Presenta il problema della nona pagina (pag. 11) che distribuisce	
		In gruppo risolvono il problema
		Espongono la soluzione trovata

Mezzi

Fotocopie da distribuire agli allievi degli appunti preparati. Può fare comodo avere una lavagna luminosa e dei lucidi con i disegni e grafici.

Tempi

Tre ore di lezione.

Materiale

Si è utilizzata, con ampie variazioni, l'unità didattica *Intro to Trig: Basic Elements of Right Triangle Geometry* a cura dell'*Office for Mathematics, Science and Technology Education - University of Illinois at Urbana Campaign* reperibile all'indirizzo Internet www.mste.uiuc.edu/tcd/Trig/intro.html.

An Introduction to Trigonometry

by Paolo Delise - I.T.C. Carli - Trieste

Remember: *there are two ways to label angles: Greek letters (α, β, γ, δ, θ, ...) or three letters with a hat over the central one:* $\widehat{ABC}$ *means an angle with two sides AB and BC and a vertex B.*

Last year we learned how to solve problems. To solve a problem we had to find:
- The data
- The unknowns
- The solving algorithm

Now, let's solve a new problem.

The Problem

Problem Description They have asked you to buy a new ladder for the Department. In order to purchase a ladder which is able to reach any building in town you need to find out its length. The highest building is ten storeys high. So, what is the length of the ladder you need to buy?
In this module the students will,
- Investigate trigonometric ratios.
- Examine the graphs of simple trig functions.
- Investigate the connections between various trig identities.

To be able to find the height of the ladder that you need to buy, it is necessary to use trig. Our problem can be described in diagram A. The diagram shows the building and a ladder leaning against it. The height of the tallest building in the town is 10 storeys[1] (remember that a story is equal to 3 m). Therefore, the building is **30 m** tall. Notice how the ground, the building and the ladder form three sides of a triangle.

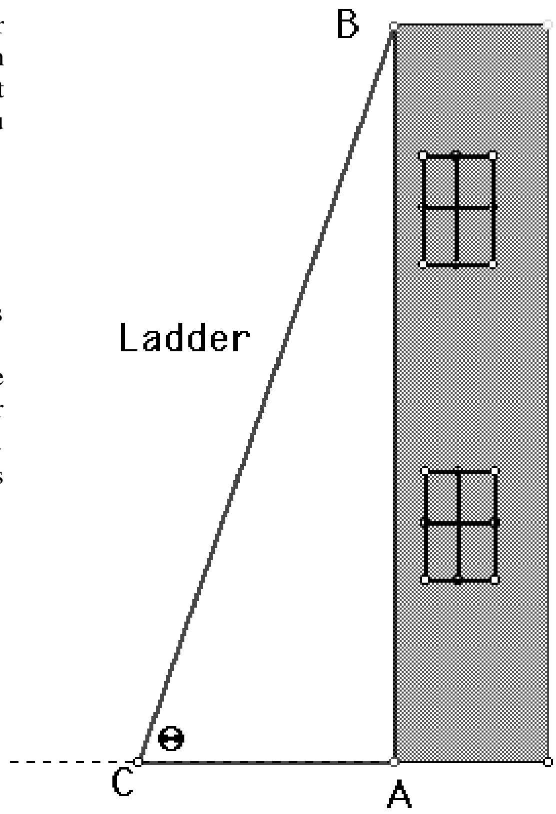


Diagram A

Questions to think about:
- Why don't we just buy a 30m ladder?
- Why don't we buy a 60 m ladder?

[1] A storey is a section of building with rooms all at the same level; synonym: floor.

Let's take a trip to the hardware store

At the store they don't sell ladders 30 m long, so we'll have to custom order it. You find out that the cost is 6 €/m. There is a label on the ladder which reads:

The ladder can only be safely used if the angle the ladder makes with the ground is not greater than 75°

See diagram B.

Notice how the ladder, building and ground make up three sides of a triangle.

The first thing we need to do is to draw the angle the ladder makes with the ground so that it measures 75°.

And what happens if you don't own a protractor[2]? Well the diagram is quite right.

We already know the measure of the angle and the length of one side (ie. the height of the building). We can now find out every angle of the triangle . Remember that the sum of the angles of a triangle is 180°. So,

$\widehat{CBA}$ = ____________

Measure AB, BC and AC. If AB = 30m

AC = ________________

and

BC (the ladder) = ___________________

What do you think of this question ?

- Why do we settle the ladder at 75° and not at 60° or 45°?

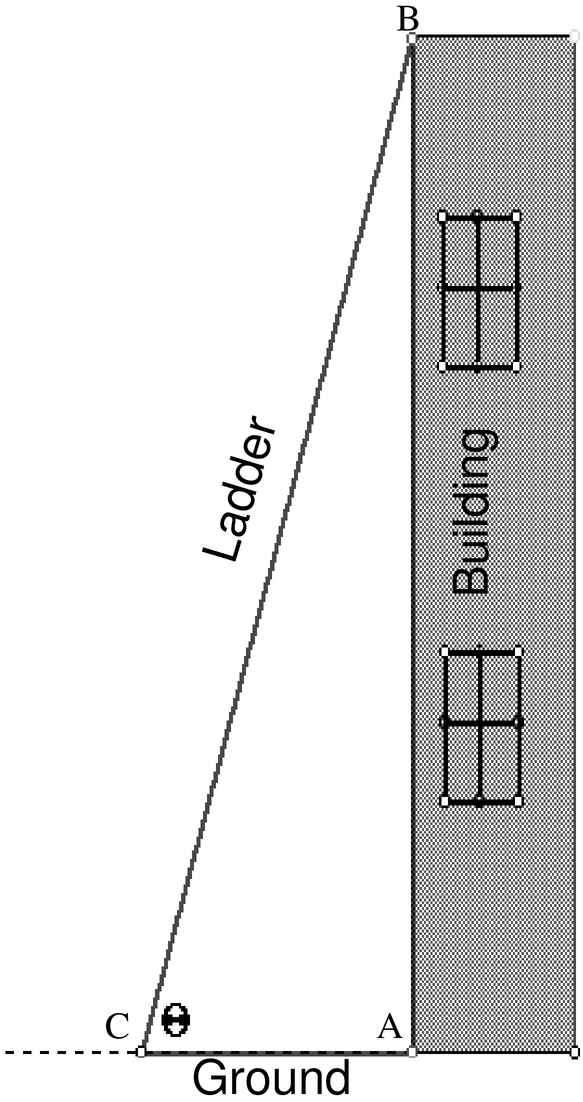


Diagram B

[2] Protractor: instrument usually in the form of a semi-circle with degrees (0° to 180°) marked on it, used for measuring and drawing angles. Synonym: goniometer.

Compute the ratio between the length of the building and the length of the ladder.

Draw a straight line C'B' parallel to CB.

notice that the ratios $\frac{BA}{BC}$ and $\frac{B'A}{B'C'}$ are ________________.

To change the value of the ratio you have to change the angle .

This ratio is called **SINE** of the angle θ. Many pocket calculators can give you the value of such a function.

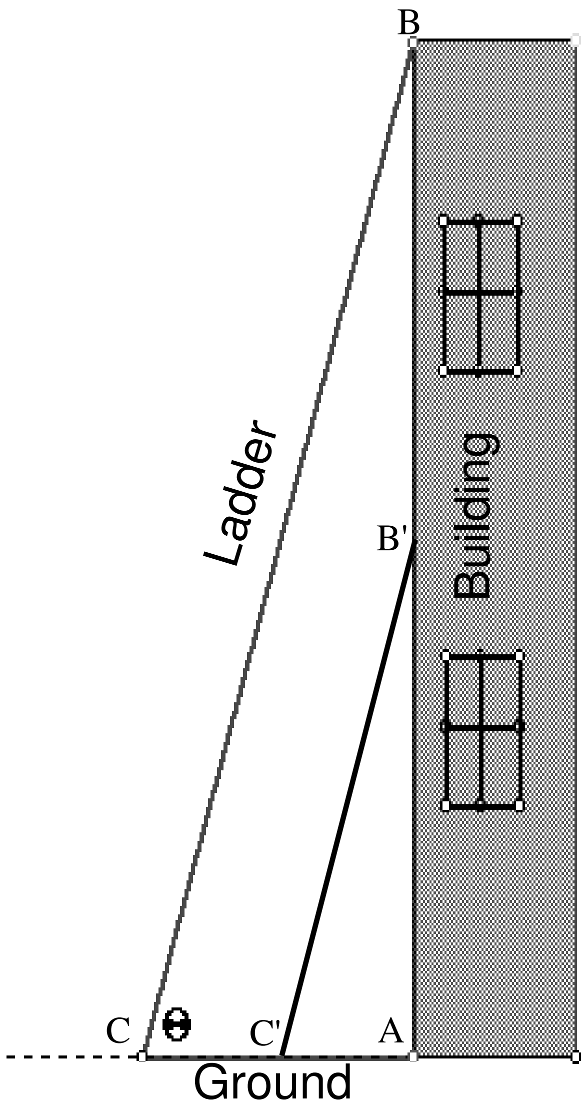
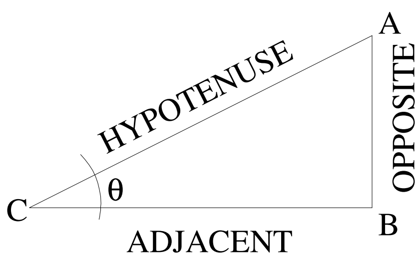


Diagram C

It is common to label the sides of a triangle as in the figure below.



NOTE that the side names are dependent on the angle you are considering except for the HYPOTENUSE which is always the side across from the right angle.

Write a ratio using the names of the sides in the picture above to describe the

SIN of $\widehat{CAB}$ = ________________________

Other trigonometric ratios (1)

Recall from the previous section that you found the SIN of an angle to be the ratio of the OPPOSITE side to the HYPOTENUSE.

As you may have thought there are other ratios computed using other pairs of sides of the triangle.

In this section, we will investigate the other major types of trig ratios.

How many ratios can you write using the three sides of the triangle? Recall combinatorics[3]. You have two "boxes" to fill, let's call them NUMERATOR and DENOMINATOR, and you have three "sides" to fill with. So[4] ....

Fill the boxes below with the ratios you have found and ask the teacher for the names of the ratios.

RATIO	NAME
OPPOSITE / HYPOTENUSE	SIN
/	
/	
/	
/	
/	
/	
/	
/	
/	

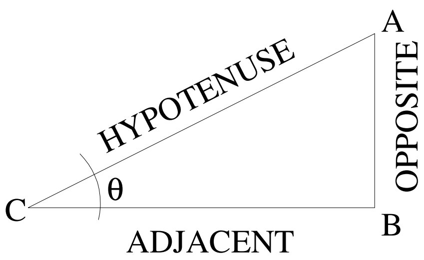
[3] i. e. combinatorial mathematics
[4] The English word is Permutation, but pay attention, please. It is not the same in Italian!

Other trigonometric ratios (2)

Fill the right column cells below and write the first column ratios using the others. TAN and COT can be written in two different ways.

RATIO		
SIN	1 / COSEC	
COS		
TAN		
COT		
SEC		
COSEC		

Now look at $\widehat{CAB}$

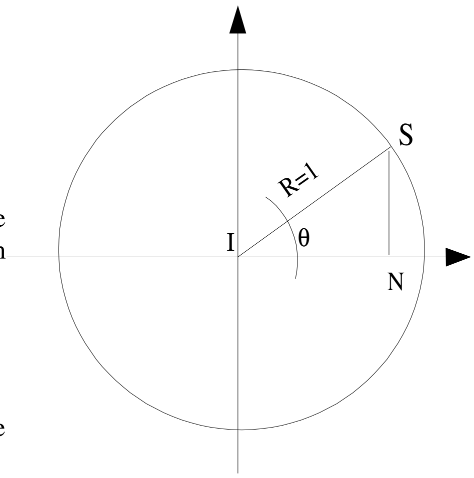


SIN of $\widehat{CAB}$ =

COS of $\widehat{CAB}$ =

TAN of $\widehat{CAB}$ =

The unit circle



Consider the diagram at the right. Let the radius of the circle be 1.

Look at triangle NSI. The HYPOTENUSE is the same as the radius of the circle. Therefore, it has a measure of 1. Recall the SIN ratio we discovered in the last section. It is,

$\sin(\theta)$ = OPPOSITE (SN) / HYPOTENUSE(IS)

Since the HYPOTENUSE = 1, the $\sin(\theta)$ = the length of the OPPOSITE side. That's what makes the UNIT CIRCLE special![5]

The same can be said for COS.

Think about the Pythagorean Theorem and how it could be used to write SIN using COS and vice-versa.

Please, try to write it and then check it with your teacher.

[5] Well, it is not really so! Don't forget SN is a length (meter) and $\sin(\widehat{SIX})$ is a ratio, i.e. a number. The two numbers are the same but they are not the same thing. My age in years (55) and the number 55 are not the same thing.

The SIN and COS functions

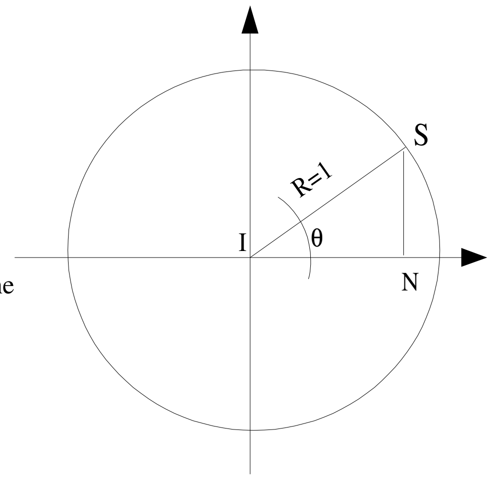
Forget now triangles and think of the unit circle.

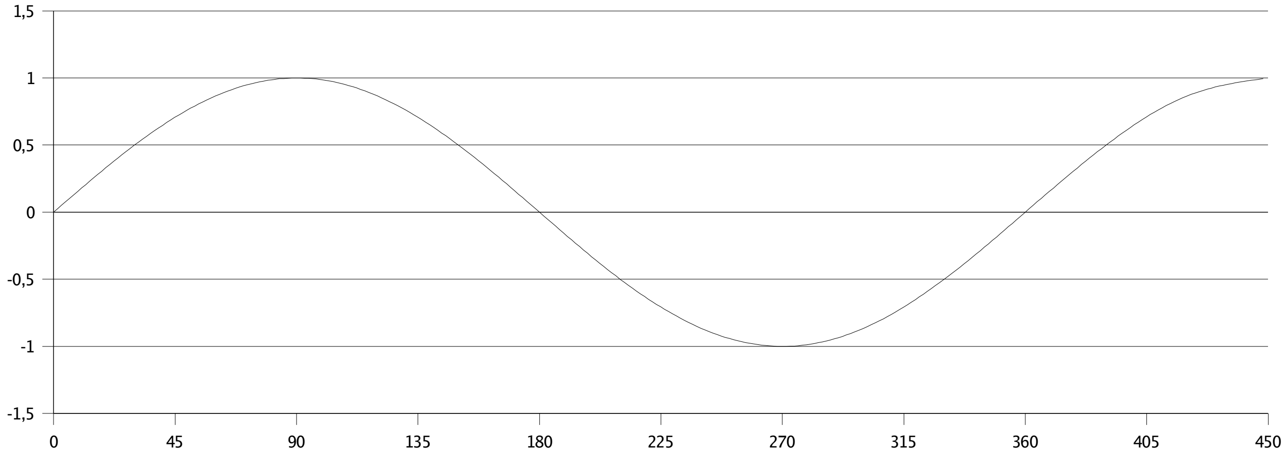
To start. let θ = 0.

Let the radius spin anticlockwise around the centre.

θ will go through 30°, 45°, 60°, 90°, 180°, 270°, 360°, 540°, 720°... The length of the opposite side is the sin(θ).

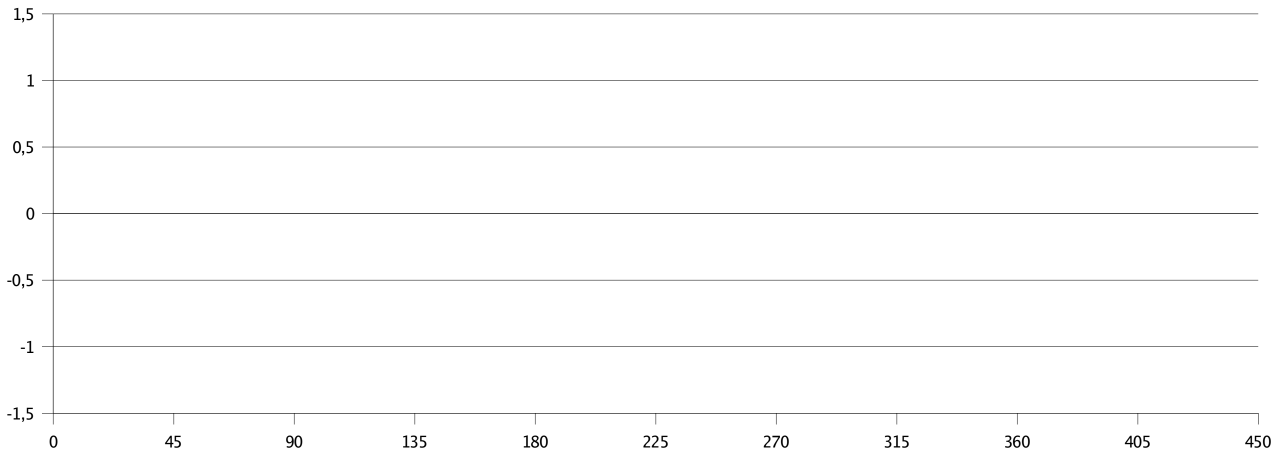
y=sin(θ) is plotted below.

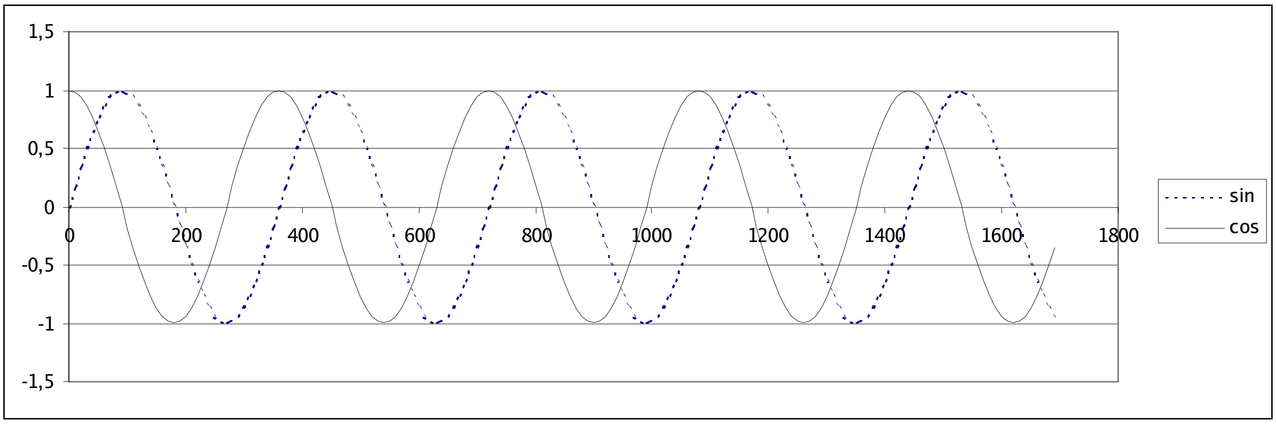




It is a periodic function (i. e. it repeats itself at regular intervals, 360°), its range is from -1 to 1.

Draw on the plot below y=cos(θ). Help yourself with a pocket calculator. Compare it to the plot above.





Does your plot looked something like this? Mathematicians and engineers say that there is a phase difference between sin and cos. The difference is 90° because the *"shape"* of COS is the same as SIN, but it has a 90° shift.

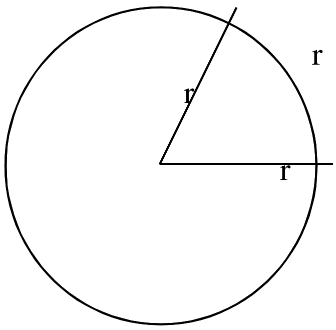
The radian

The last question

Why the unit of measure for the angles is such that the right angle is 90°? Why not 100°? why not 25° and the straight angle 50° and the round angle 100°?

The answer is hidden in the history of maths. Long and long ago, when there were no pocket calculators (30 years ago), no logs (500 years ago), when people used beads[6] and abacuses (up to 4.000 years ago) to compute, dividing two numbers was a very hard problem to solve. And you can easily divide 360 by 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 36, ... and even 360 : 16 is rather easy. On the contrary you cannot divide easily 100 or 400 by 3, 6, and so on. For the same reason the day is 12 hours long (and 12 hours the night, but this is another problem)[7].

When mathematicians learned to divide two relatively prime numbers, they looked for a better way to measure angles. They found it was rather comfortable to make use of the so called radian measure of angles. The unit of measure is the acute central angle subtended[8] in a circle by an arc whose length is equal to the radius of the circle. Such an angle is termed a RADIAN.



The measure of a round angle is 2π radians, a flat angle is π radians, a right angle is $\pi/2$ radians, and mathematicians ***love*** π!

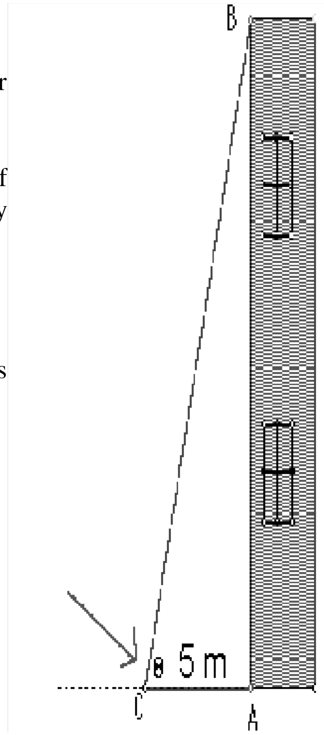
The introduction of radian measure of angles permits casting many formulas in simpler forms.

6 Bead is a small peace of hard material with a hole trough it.

7 Ancient Romans used to divide the daylight time in twelve hours; the hours were longer in summer and shorter in winter. They divided the night time in 4 *"vigiliae"* i. e. 4 night guard shifts.

8 *Subtended* means *opposite to.*

The last problem



Try to solve this problem, now. Try to solve it by yourself or, better, together with your school mates, but speak English please!

You are in front of the highest building of the town and you want to measure the height of the building. You want a positive proof it is 30 m high, but you cannot measure it. By chance you have a tape-measure[9] 5 m long and a protractor (oh!) in your pockets!

Can you compute the height of the building?

N.B. Surveyors[10] do not use protractors to do such a job; they use theodolites, but it was very unlikely to find a theodolite into your pockets, even more unlikely than a protractor!

This document was written using OpenOffice.org 1.0.2 and Linux

[9] A *tape-measure*, also tape or measuring tape, is a tape of flexible metal marked in centimetres for measuring length.

[10] A *surveyor* is a person officially engaged in examining a building. In Trieste there is a school for surveyors. Its name is "Max Fabiani".